Rational and Irrational Numbers

Goal: Work with irrational numbers.

Vocabulary

Irrational number: A number that cannot be written as a quotient of two integers

Real number: Any number in the set of all rational numbers and irrational numbers

EXAMPLE 1 — Classifying Real Numbers

Tell whether the number is *rational* or *irrational*.

a. $\frac{4}{5}$ b. 0.31818...   c. $\sqrt{2}$

Solution

a. Because $\boxed{\frac{4}{5}}$ is the quotient of two integers (4 and 5), it is rational.

b. Because   0.31818... is a repeating decimal, it is rational.

c. Because 2 is a positive integer and is not a perfect square, $\boxed{\sqrt{2}}$ is irrational.

**Guided Practice**   Tell whether the number is *rational* or *irrational*. **Explain your reasoning.**

1. $\frac{9}{20}$	2. $\sqrt{13}$	3. $\frac{5}{6}$	4. $\sqrt{81}$

EXAMPLE 2 **Comparing Real Numbers**

Graph $\sqrt{\frac{2}{3}}$ and $\frac{2}{3}$ on a number line. Then copy and complete the statement $\sqrt{\frac{2}{3}}$ _?_ $\frac{2}{3}$ with < or >.
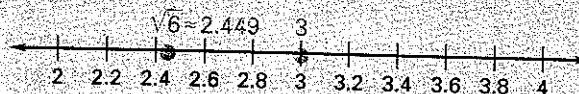
Solution

Use a calculator to approximate the square root and write any fractions as decimals. Then graph the numbers on a number line and compare.
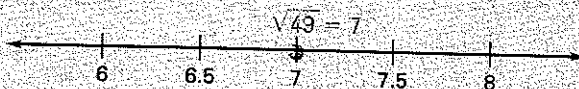

$\frac{2}{3} = 0.\overline{6}$ $\sqrt{\frac{2}{3}} \approx 0.8165$

0   0.1   0.2   0.3   0.4   0.5   0.6   0.7   0.8   0.9   1
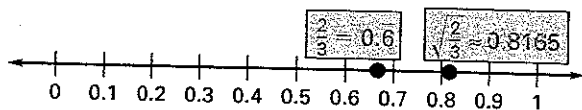
Answer: Because $\sqrt{\frac{2}{3}}$ is [to the right] of $\frac{2}{3}$, $\sqrt{\frac{2}{3}}$ [>] $\frac{2}{3}$.

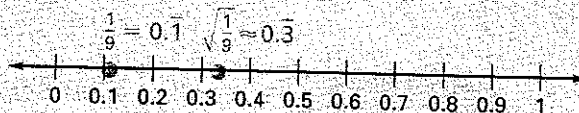**Guided Practice** Graph the pair of numbers on a number line. Then copy and complete the statement with <, >, or =.

5. $\sqrt{6}$ _?_ 3

$\sqrt{6} \approx 2.449$ 3

2   2.2   2.4   2.6   2.8   3   3.2   3.4   3.6   3.8   4

6. $\sqrt{49}$ _?_ 7

$\sqrt{49} = 7$

6   6.5   7   7.5   8

7. $\sqrt{\frac{1}{9}}$ _?_ $\frac{1}{9}$

$\frac{1}{9} = 0.\overline{1}$ $\sqrt{\frac{1}{9}} \approx 0.\overline{3}$

0   0.1   0.2   0.3   0.4   0.5   0.6   0.7   0.8   0.9   1

EXAMPLE 3 **Ordering Decimals**

Order the decimals $0.\overline{63}$, 0.633, $0.6\overline{3}$, and $0.\overline{636}$ from least to greatest.

Step 1 Write each decimal out to six decimal places.

$0.\overline{63}$ = 0.636363... 0.633 = 0.633000...

$0.6\overline{3}$ = 0.633333... $0.\overline{636}$ = 0.636636...

step 2 Write the decimals in order from least to greatest,

0.633000... , 0.633333... , 0.636363... , 0.636636...

Answer: From least to greatest, the order is

0.633 , 0.6$\overline{3}$, 0.$\overline{63}$, and 0.$\overline{636}$.

EXAMPLE 4 **Using an Irrational Number in Real Life**

Stopping Distance The equation $s = \sqrt{27d}$ gives the minimum distance that it takes a car to stop on dry pavement after applying the brakes. In the equation, d represents the stopping distance in feet of a car that is traveling at a speed of s miles per hour. A car's stopping distance is 138 feet. How fast was the car traveling?

Solution

$s = \sqrt{27d}$ Write original equation.

$= \sqrt{27(138)}$ Substitute 138 for d.

$= \sqrt{3726}$ Multiply.

≈ 61.04 Approximate square root.

Answer: The car was traveling about 61.04 miles per hour.

Homework

Guided Practice Use the equation in Example 4.

8. A car's stopping distance is 200 feet. How fast was the car traveling?